Saint Ignatius' College, Riverview
Mathematics Assessment Task
2021

Year 12
Mathematics (Extension One)
Task 4
Date: 20th August 2021

General Instructions:

- **Reading time: 5 minutes**
- **Time Allowed: 1.5 hours**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and **your teacher's code** on each writing page
- Marks may not be awarded for missing or carelessly arranged working.

Teachers:

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr N Mushan **NHM**
- Mr P Collins **PPC**

Topics Examined:

Short Answer

Question 1	**15 Marks**
Question 2	**15 Marks**
Question 3	**15 Marks**
Question 4	**15 Marks**
Total	**60 Marks**

Question 1 (START A NEW WRITING PAGE)

(a) Consider the polynomial $P(x) = x^3 - x^2 - 8x + 12$.

(i) Use the factor theorem to show that $(x-2)$ is a factor of $P(x)$. **(1)**

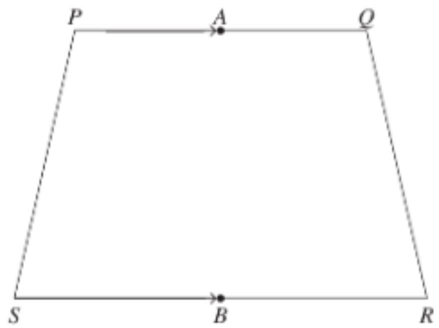
(ii) Factorise $P(x)$ completely. **(2)**

(b) Solve the equation $\sin 2x = \tan x$ for the domain $[0, \pi)$ **(3)**

(c) Prove $n(n+2)$ is divisible by 4 by mathematical induction, **(3)**
if n is any positive **even** integer.

(d) Find the value of k if $y = \tan x$ satisfies the differential equation $y' = k + y^2$. **(2)**

(e) PQRS is a trapezium with A and B being the midpoints of PQ and RS respectively.



Let $\overline{PA} = \underset{\sim}{a}$ and $\overline{SB} = \underset{\sim}{b}$

(i) Show that $\overline{QR} = \underset{\sim}{b} - \underset{\sim}{a} + \overline{AB}$ **(1)**

(ii) Express $\overline{PS}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\overline{AB}$. **(1)**

(iii) Write an expression for $\overline{PS} + \overline{QR}$, in terms of $\overline{AB}$ **(2)**

END OF QUESTION 1

QUESTION 2 (START A NEW WRITING PAGE)

(a) (i) Write the expression $\sin x+\sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$, where $R>0$. and $0\le x\le\frac{\pi}{2}$ **(2)**

(ii) Hence, solve the equation $\sin x+\sqrt{3}\cos x=1$ in the domain $[0,2\pi]$ **(2)**

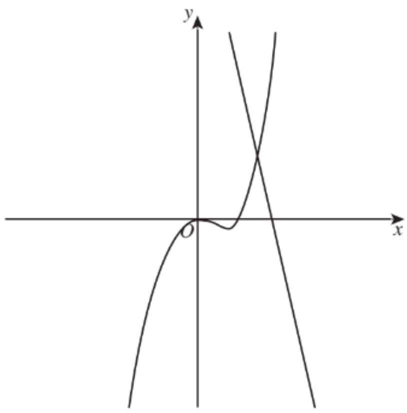
(b) After time t years from the start of the year 2021, the number of people in a population is given by:

$N=70+Ae^{0.1t}$ where A is a constant greater than 0.

(i) Show that $\frac{dN}{dt}=0.1(N-70)$. **(1)**

(ii) There were 100 people in the population at the start of the year 2021. Find the year when the population size will exceed 190. **(3)**

(c) The graphs of the functions $g(x)=mx+b$ and $f(x)=3x^3-2x^2$ are shown.



(i) Sketch the graph of $y=3|x|^3-2|x|^2$ **(1)**

(ii) Use the sketch of $y=3|x|^3-2|x|^2$, to find the values of m and b if the function of $g(x)=mx+b$, has the solution to the equation $mx+b<3|x|^3-2|x|^2$ is $x<-3$ or $x>1$. **(2)**

QUESTION 2 continues the next page…

(d) A NSW census shows that 40% of NSW adults completed at least 30 minutes of exercise each day.

(i) A random sample of 21 NSW adults is to be conducted to find the proportion who completed at least 30 minutes of exercise each day. Given that the mean is 0.4, show that standard deviation for the distribution of sample proportions 0.1069. **(2)**

(ii) Part of a table giving values of $P(Z \le z)$, where z is a standard normal variable is shown below **(2)**

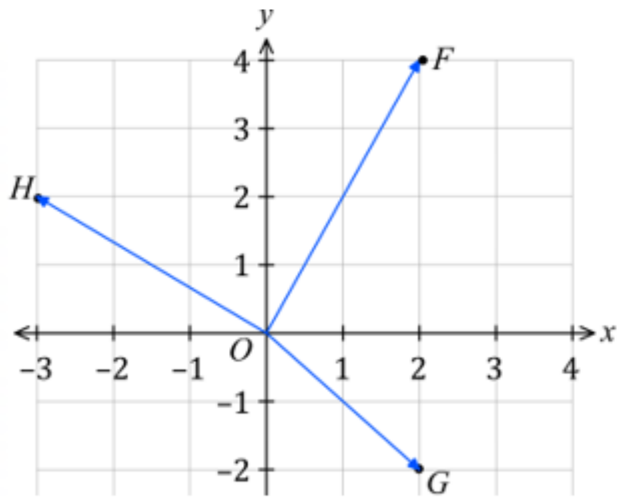
z	*+0.00*	*+0.01*	*+0.02*	*+0.03*	*+0.04*	*+0.05*	*+0.06*	*+0.07*	*+0.08*	*+0.09*
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9931	0.9934	0.9936
2.5	0.9938	0.9940	0.9943	0.9945	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

Estimate the probability that a random sample of 21 NSW adults find at most 3 who have completed at least 30 minutes of exercise each day.

END OF QUESTION 2

QUESTION 3 (START A NEW WRITING PAGE)

(a) The vectors $\overline{OF}$, $\overline{OG}$ and $\overline{OH}$ are shown below.



Not to scale

What is the size of $\angle GOH$ to the nearest degree? **(2)**

(b) What is the derivative of $\sin x.\cos^{-1} x$? **(2)**

(c) Use the substitution $u = \cos 2\theta$ to evaluate $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos^2 2\theta \sin 2\theta \, d\theta.$ **(2)**

(d) A spherical balloon is to be filled with water so that its surface area increases at a constant rate of 1cm^2/s.

[Note: Sphere formulae $SA = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$]

(i) Find when the radius is 3 cm:

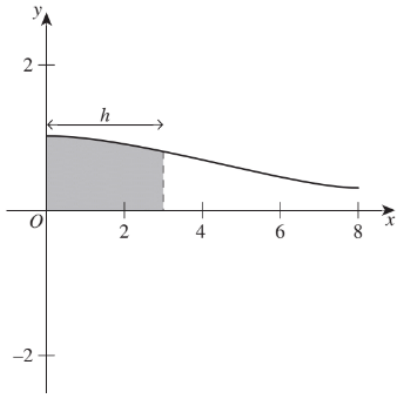
(α) the required rate of increase of the radius; **(2)**
(leave your answer in terms of π)

(β) the rate at which the water is flowing in at that time. **(1)**

(ii) Find the volume when the volume is increasing at 10 cm^3/s **(2)**
(leave your answer in terms of π)

QUESTION 3 continues the next page…

(e) The graph of the function $y = \dfrac{3}{\sqrt{9+x^2}}$ for $x > 0$ is shown.



The area bounded by the curve $y = \dfrac{3}{\sqrt{9+x^2}}$, the axes and the lines $x=0$ **(2)**

and $x = h$ is rotated about the *x*-axis to create a solid of revolution.

Find in terms of *h*, the volume of this solid.

(f) The spread of flu through a student population is modelled by the equation

$$S = \frac{2000}{1+199e^{-0.4t}}$$

where S is the total number of students infected after t days.

Show that the given equation for S satisfies the differential equation **(2)**

$$\frac{dS}{dt} = \frac{S}{5}\left(2 - \frac{S}{1000}\right).$$

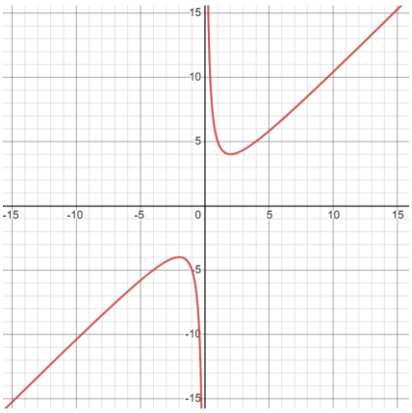
END OF QUESTION 3

QUESTION 4 (START A NEW WRITING PAGE)

(a) Show that $\displaystyle\int_0^{\frac{\pi}{4}} (1+\tan x)^2 \, dx = \ln 2 + 1$ **(3)**

(b) Prove that $\dfrac{\cos 3\theta}{\cos\theta} - \dfrac{\sin 3\theta}{\sin\theta}$ is independent of θ. **(2)**

(c) The graph of the function $f(x) = \dfrac{x^2+4}{x}$ is shown.



(i) Show that the stationary points of $f(x) = \dfrac{x^2+4}{x}$ are (-2, -4) and (2, 4). **(2)**

(ii) Sketch the graph of $y = \dfrac{1}{\sqrt{f(x)}}$, **(2)**

showing all important features including turning point(s), intercept(s) and asymptotes.

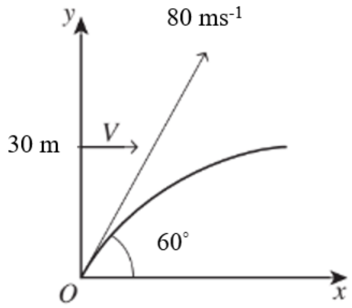
QUESTION 4 continues the next page…

(d) Initially, a golf ball was hit from the ground at a velocity of 80 ms^{-1} at an angle of 60° to the horizontal and $g = 10\,m\,s^{-2}$.

The position vector of the golf ball after t seconds is given by

$\underset{\sim}{s}(t) = (40t)\underset{\sim}{i} + \left(40\sqrt{3}t - 5t^2\right)\underset{\sim}{j}$.

(i) Eight seconds **after** the golf ball was hit, a small stone was fired at a velocity V **horizontally** from a point 30 m above the ground.



Find the position vector of the stone. **(2)**

(ii) Find the time to the nearest second after the golf ball was hit when it collided with the stone. **(2)**

(iii) What is the stone's speed at collision? **(2)**

(Give your answer to three significant figures)

END OF QUESTION 4

END OF EXAMINATION

SUGGESTED SOLUTIONS

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2021 SIC EXT 1 FINAL

Q1 (a) $P(x) = x^3 - x^2 - 8x + 12$

(i) $P(2) = 2^3 - 2^2 - 8(2) + 12$
$= 8 - 4 - 16 + 12$ — either seen 1 Mark
$= 0$
$\therefore (x-2)$ is a factor.

(ii) $P(-3) = (-3)^3 - (-3)^2 - 8(-3) + 12$ — Various methods available
$= -27 - 9 + 24 + 12$
$= 0$
$\therefore (x+3)$ is a factor.

Now, $\alpha\beta\gamma = -12$
$2(-3)\gamma = -12$
$\gamma = 2$
$\therefore P(x) = (x-2)^2(x+3)$

Mark 2. CAO
Mark 1. Second factor identified.

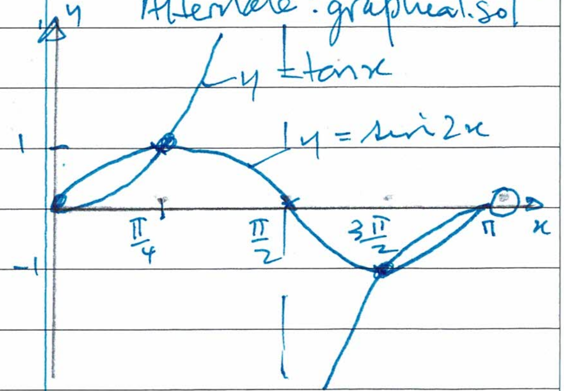
(b) $\sin 2x = \tan x$ $[0, \pi)$

$2\sin x\cos x = \dfrac{\sin x}{\cos x}$

$\therefore 2\sin x\cos^2 x - \sin x = 0$
$\sin x(2\cos^2 x - 1) = 0$
$\therefore \sin x = 0$ or $2\cos^2 x - 1 = 0$
$x = 0$ $\quad\cos^2 x = \frac{1}{2}$
$\cos x = \pm\frac{1}{\sqrt{2}}$
$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$x = 0, \frac{\pi}{4}, \frac{3\pi}{4}$ ✓

Alternate: graphical sol^n



Mark 3: Three correct solutions
Mark 2: Considerable progress ($\sin x = 0$ missing)
Mark 1, Some progress made.

(c) $n(n+2) = 4M$ (M an integer)

1. Prove true for $n = 2$
$2(2+2) = 4 \times 2$ $\therefore$ true for $n=2$

Mark 1: Steps 1+2

2. Assume true for $n = K$
i.e. $K(K+2) = 4N$ (N is an integer)

Mark 2: Set up in $K+2$

3. Prove true for $n = K+2$
i.e. $(K+2)((K+2)+2) = 4P$ (P an integer)

LHS $= (K+2)(K+4)$
$= K(K+2) + 4(K+2)$

Mark 3: Substitution and resolution

(continued over)

$$LHS = K(K+2) + 4(K+2)$$
$$= 4N + 4(K+2)$$
$$= 4(N+K+2)$$ as N, K and 2 are all integer $(N+K+2)$ is an integer.
$$= 4P$$
$$= RHS$$

$\therefore$ true for $n = K+2$ if true for $n = K$. As true for $n = 2$, true for $n = 4$ and so on.

$\therefore$ True for all positive even numbers.

Problems: let $n = 2K$ many did not define K, if K is even then even integers are missed

- leaving the substitution with K in the denominator – is the term an integer eg. $\frac{8}{6} = \frac{4}{3}$ a non-integer.

(d) $y = \tan x$

$y' = \sec^2 x$

$\therefore \sec^2 x = K + \tan^2 x$

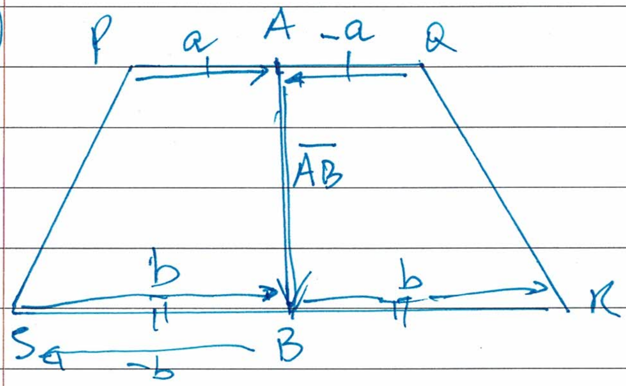
Pythag: $\sec^2 x = 1 + \tan^2 x$

$\therefore K = 1$

Mark 1: $y' = \sec^2 x$

Mark 2: $K = 1$

(e)



YOU SHOULD HAVE A DIAGRAM AS PART OF YOUR ANSWER!!

(i) $\overline{QR} = -a + \overline{AB} + b$

$= \underset{\sim}{b} - \underset{\sim}{a} + \overline{AB}$

(i) Diagram or notation

$\overline{QR} = \overline{QA} + \overline{AB} + \overline{BR}$

$= -a + \overline{AB} + b$

(ii) $\overline{PS} = \underset{\sim}{a} + \overline{AB} - \underset{\sim}{b}$

$= -\underset{\sim}{b} + \underset{\sim}{a} + \overline{AB}$

1 Mark

(iii) $\overline{PS} + \overline{QR} = -\underset{\sim}{b} + \underset{\sim}{a} + \overline{AB} + \underset{\sim}{b} - \underset{\sim}{a} + \overline{AB}$

$= 2\overline{AB}$

Mark 2: $2\overline{AB}$

Mark 1: attempted addition

STION TWO

a) i) $\sin x + \sqrt{3}\cos x = R\sin(x+\alpha)$ — Well done

Now: $R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$ — Working was needed.

$\therefore \sin x + \sqrt{3}\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$

if $R\cos\alpha = 1$ $\quad \therefore \tan\alpha = \sqrt{3}$

$R\sin\alpha = \sqrt{3}$ $\quad \alpha = \frac{\pi}{3}$ ← 1mk

$R^2\cos^2\alpha + R^2\sin^2\alpha = 1+3$

$R^2(\cos^2\alpha + \sin^2\alpha) = 4 \quad \therefore R = 2$ ← 1mk

So $\sin x + \sqrt{3}\cos x = 2\sin\left(x + \frac{\pi}{3}\right)$

ii) $2\sin\left(x + \frac{\pi}{3}\right) = 1 \quad [0, 2\pi]$

$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$ — Well done.

$\therefore x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \ldots$

$x = -\frac{\pi}{6}, \frac{\pi}{2}, \frac{11\pi}{6}, \ldots$

$\therefore$ For $[0, 2\pi]$

$x = \frac{\pi}{2}, \frac{11\pi}{6}$ ← 1mk for each

b) i) $N = 70 + Ae^{0.1t}$

$$\frac{dN}{dt} = 0.1\,Ae^{0.1t}.$$

Note: $Ae^{0.1t} = N - 70$

1mk for EITHER.

$$\therefore \frac{dN}{dt} = 0.1(N-70)$$

ii) $N = 70 + A\,e^{0.1t}$

Well done

$t = 0 \quad N = 100$

$\therefore 100 = 70\ A\,e^{0}$

$\therefore A = 30$ ← 1mk

$N = 70 + 30e^{0.1t}$

Let $N = 190$

$190 = 70 + 30e^{0.1t}$ ← 1mk

$4 = e^{0.1t}$

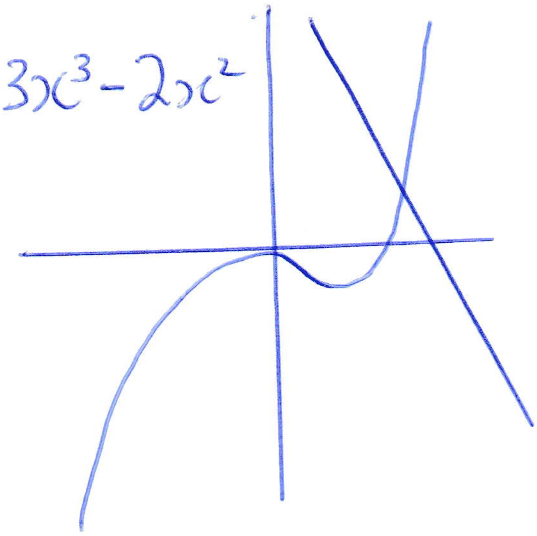
$$\therefore t = \frac{\ln 4}{0.1}$$

$= 13.8$ yrs

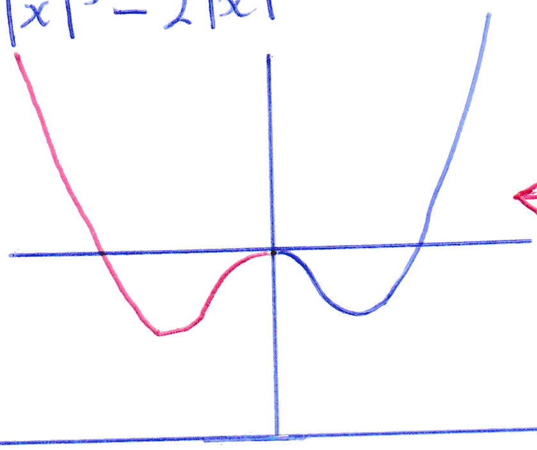
ie Year 2034

1mk

c) $y = 3x^3 - 2x^2$



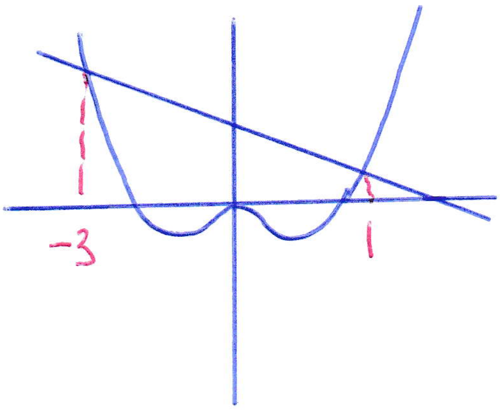
i) $y = 3|x|^3 - 2|x|^2$



Many struggled.

1mk

ii)



$x = -3$
$y = 63$ $(-3, 63)$

$x = 1$
$y = 1$ $(1, 1)$.

Poorly done

1mk for correct m = and b =

"Part marks awarded for some progress"

$$m = \frac{63-1}{-3-1} = -\frac{31}{2} = -15.5$$

$y = \frac{-31}{2}x + b$

$(1, 1)$

$1 = \frac{-31}{2} + b$

$b = \frac{33}{2} = 16.5$

d) i) $p = 0.4$ $n = 21$

$\sigma = \sqrt{VAR}$

$= \sqrt{\frac{0.4(1-0.4)}{21}}$

$\doteqdot 0.1069$

A number of methods used

2mks

ii) $Z = \frac{x - \mu}{\sigma}$ $x = \frac{3}{21}$

$= 0.1428$

$Z = \frac{0.1428 - 0.4}{0.1069}$

$= -2.41$

1mk

A z score of 2.41 gives a probability of 0.9920

$\therefore$ for $Z = -2.41$

Prob $= 0.008$

Too many misread the table

1mk

Month
1 2 3 4 5 6
7 8 9 10 11 12

Date ☐Mon ☐Tue ☐Wed ☐Thu ☐Fri ☐Sat ☐Sun

Yr 12 Extension One Maths Exam (Trial)

Q3.

(a) $\overrightarrow{OH} = -3\underset{\sim}{i} + 2\underset{\sim}{j}$ $\quad |\overrightarrow{OH}| = \sqrt{13}$

$\overrightarrow{OG} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$ $\quad |\overrightarrow{OG}| = \sqrt{8}$

$$\frac{\overrightarrow{OH} \cdot \overrightarrow{OG}}{|\overrightarrow{OH}||\overrightarrow{OG}|} = \cos\angle GOH$$ ✓

(Well Answered)

$$\frac{-6-4}{\sqrt{13}\sqrt{8}} = \cos\angle GOH$$

$$\angle GOH = \cos^{-1}\frac{-10}{\sqrt{104}} = 168.69^\circ = 169^\circ$$ ✓

b) $y = \sin x \cos^{-1} x$ $\quad y = u \times v$

$y' = uv' + vu'$ ✓

(Well Answered)

$$y' = (\sin x)\left(\frac{-1}{\sqrt{1-x^2}}\right) + \cos x \cos^{-1} x$$

$$y' = \frac{-\sin x}{\sqrt{1-x^2}} + \cos x \cos^{-1} x$$ ✓

(Some students changed $\cos x \cos^{-1} x$ to x or 1 which is incorrect)

c) $u = \cos 2\theta$ $\quad \theta = \frac{3\pi}{4}$ $\quad u = \cos\frac{3\pi}{2} = 0$

$\frac{du}{d\theta} = -2\sin 2\theta$

$\theta = \frac{\pi}{2}$ $\quad u = \cos \pi = -1$

$du = -2\sin 2\theta \, d\theta$ $\quad -\frac{1}{2}du = \sin 2\theta \, d\theta$

(* Some students forgot to include 2 in derivative here)

$$I = \frac{1}{-2}\int_{-1}^{0} u^2 \, du$$

Well Answered

$$I = \frac{1}{2}\int_{0}^{-1} u^2 \, du$$

$$I = \frac{1}{2}\left[\frac{u^3}{3}\right]_0^{-1}$$

$$I = -\frac{1}{6}$$

(Some students made mistakes calculating in the last step)

d)

(i)

a) $\frac{dA}{dt} = 1 \text{ cm}^2/\text{s}.$ $\quad \frac{dr}{dt} = ?$ when $r = 3$cm.

$$\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA}$$

Month
1 2 3 4 5 6
7 8 9 10 11 12

Date ☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r \qquad \frac{dr}{dA} = \frac{1}{8\pi r}.$

Well answered

$\frac{dr}{dt} = 1 \times \frac{1}{8\pi r} = \frac{1}{8\pi r}$ cm/s. $= \frac{1}{24\pi}$

b) $\frac{dV}{dt} = ? \qquad r = 3$ cm.

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

Well answered

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 3\left(\frac{4}{3}\right)\pi r^2 = 4\pi r^2$ when $r = 3$

$\frac{dV}{dr} = 36\pi$

$\frac{dV}{dt} = 36\pi \times \frac{1}{8\pi(3)} = \frac{36}{24} = \frac{3}{2} = 1{\cdot}5$ cm^3/s.

(ii) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}.$

Many students assumed $r=3$ here

$10 = 4\pi r^2 \times \frac{1}{8\pi r} \qquad 10 = \frac{4r^2}{8r}$

$80 = 4r \qquad r = 20$

$V = \frac{4}{3}\pi(20)^3 = \frac{32000\pi}{3}$ cm^3

e) $y = \dfrac{3}{\sqrt{9+x^2}}$

$$V = \pi \int_0^h y^2 \, dx.$$

(Well Answered)

$$V = \pi \int_0^h \frac{9}{9+x^2}$$

$$V = 9\pi \int_0^h \frac{1}{3^2+x^2} \quad \checkmark$$

$$V = 9\pi \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_0^h$$

$$V = 9\pi \left(\frac{1}{3} \tan^{-1} \frac{h}{3} \right) - 9\pi \left(\frac{1}{3} \tan^{-1} 0 \right)$$

$$V = 3\pi \tan^{-1} \frac{h}{3}. \quad \checkmark$$

Month
1 2 3 4 5 6
7 8 9 10 11 12

Date ____________

☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun

f) $S = \dfrac{2000}{1+199e^{-0.4t}}$

L.H.S

$\dfrac{dS}{dt}$

$S = \dfrac{2000}{1+199e^{-0.4t}}$

$S = 2000(1+199e^{-0.4t})^{-1}$

$\dfrac{dS}{dt} = -2000(1+199e^{-0.4t})^{-2}(-0.4 \times 199 \times e^{-0.4t})$

$\dfrac{dS}{dt} = \dfrac{(-2000)}{(1+199e^{-0.4t})^2}\dfrac{(-79.6e^{-0.4t})}{(1)} =$

$\dfrac{dS}{dt} = \dfrac{159200e^{-0.4t}}{(1+199e^{-0.4t})^2}$

R.H.S.

$\dfrac{S}{5}\left(2 - \dfrac{S}{1000}\right) =$

$\dfrac{S}{5}\left(\dfrac{2000 - S}{1000}\right) =$

$\dfrac{S(2000-S)}{5000}$

$$\frac{\frac{2000}{1+199e^{-0.4t}}\left(2000 - \frac{2000}{1+199e^{-0.4t}}\right)}{5000}$$

$\cancel{\dfrac{2000}{1+199e^{-0.4t}}}\left(\dfrac{2000(1+199e^{-0.4t}) - 2000}{1+199e^{-0.4t}}\right) =$

$\cancel{\dfrac{2000^2 + 2000(199e^{-0.4t}) - 2000^2}{(1+199e^{-0.4t})^2}}$

(Poorly Answered)

(You need to work down L.H.S & R.H.S separately and show they are equal)

$\dfrac{2000(2000(1+199e^{-0.4t}) - 2000)}{5000(1+199e^{-0.4t})^2} = \dfrac{(2000)^2\, 199e^{-0.4t}}{5000(1+199e^{-0.4t})^2}$

$= \dfrac{159200e^{-0.4t}}{(1+199e^{-0.4t})^2}$

Q4

(a) $\displaystyle\int_0^{\frac{\pi}{4}} (1+\tan x)^2 \, dx$

$= \displaystyle\int_0^{\frac{\pi}{4}} 1 + 2\tan x + \tan^2 x \quad dx$

$= \displaystyle\int_0^{\frac{\pi}{4}} 1 + 2\,\frac{\sin x}{\cos x} + \sec^2 x - 1 \quad dx$

$= 2\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} + \sec^2 x \quad dx$ ✓ — 1 Mark

$= \left[-2\ln(\cos x) + \tan x\right]_0^{\frac{\pi}{4}}$ ✓ — 1 Mark

$= -2\ln\frac{1}{\sqrt{2}} + 1 - (-2\ln 1 + 0)$

$= 1 - 2\ln 2^{-\frac{1}{2}}$ ✓ — 1 Mark

$= 1 + \ln 2$

(b) $\dfrac{\cos 3\theta}{\cos\theta} - \dfrac{\sin 3\theta}{\sin\theta}$

$= \dfrac{\sin\theta\cos 3\theta - \cos\theta\sin 3\theta}{\cos\theta\sin\theta}$

$= \dfrac{\sin(-2\theta)}{\frac{1}{2}\,2\sin\theta\cos\theta}$ ✓ — 1 Mark

$= \dfrac{-2\sin 2\theta}{\sin 2\theta}$

$= -2$ ✓ — 1 Mark

(c) $f(x) = \dfrac{x^2+4}{x}$

(i)

$$f'(x) = \frac{x(2x) - (x^2+4)}{x^2}$$

$$= \frac{x^2-4}{x^2} \quad \checkmark \text{ — 1 mark}$$

$= 0$ at SP's

$\therefore (x-2)(x+2) = 0$ ✓

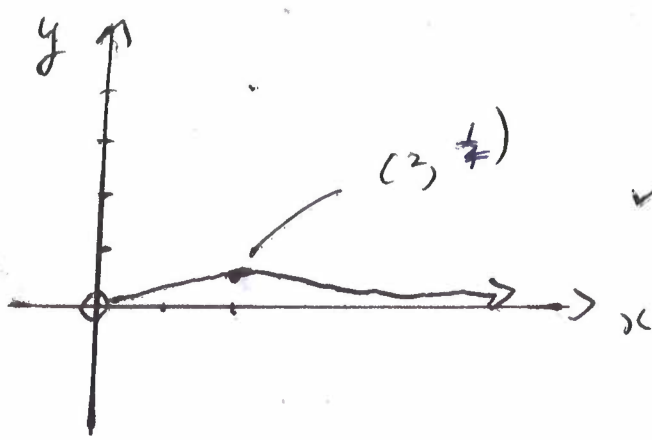
$x = \pm 2$ → 1 mark.

$y = \begin{matrix} 4 \\ -4 \end{matrix}$

SPs are $(2,4)$ and $(-2,-4)$

(ii) $y = \dfrac{1}{\sqrt{f(x)}}$

$\therefore f(x) > 0$ ✓ — 1 mark



✓ — correct stat point identity

NOTE: should be open circle at origin but I accepted open/closed

(d) $\underset{\sim}{s}(t) = (40t)\underset{\sim}{i} + (40\sqrt{3}t - 5t^2)\underset{\sim}{j}$

(i) $\underset{\sim}{a}(t) = -10\underset{\sim}{j}$

$\underset{\sim}{v}(t) = -10t\underset{\sim}{j} + c$

$t=0 \quad \underset{\sim}{v}(t) = V\underset{\sim}{i}$

$\therefore \quad c = V\underset{\sim}{i}$

$\underset{\sim}{v}(t) = V\underset{\sim}{i} + (-10t)\underset{\sim}{j}$ ✓

$\underset{\sim}{s}(t) = Vt\underset{\sim}{i} + (-5t^2)\underset{\sim}{j} + c$

$t=0 \quad \underset{\sim}{s}(t) = 30\underset{\sim}{j}$

$\therefore c = 30\underset{\sim}{j}$

$s(t) = Vt\underset{\sim}{i} + (-5t^2 + 30)\underset{\sim}{j}$ ✓

* NOTE! Can also have answer as $V(t-8)\underset{\sim}{i} + (-5(t-8)^2 + 30)\underset{\sim}{j}$

* Could also have

$s(t) = V(t-8)\underset{\sim}{i} + (-5(t-8)^2 + 30)\underset{\sim}{j}$

(ii) When they collide

$40t = v(t-8) \cdots ①$ and $40\sqrt{3}t - 5t^2 = 30 - 5(t-8)^2 \cdots ②$

Using ②

$$40\sqrt{3}t - 5t^2 = 30 - 5t^2 + 80t - 320 \checkmark$$

$$290 = 80t - 40\sqrt{3}t$$

$$t = \frac{290}{80 - 40\sqrt{3}} \doteqdot 27.057$$

$$\therefore \doteqdot 27 \checkmark$$

* NOTE! Poorly done as many students did <u>not</u> account for the time difference - no marks awarded

(iii) Need to find the initial velocity of stone, V.

from part (ii)

$40t = V(t-8)$ when $t \doteqdot 27$

$\therefore 40 \times 27 = V \times (27-8)$

$V = \frac{40 \times 27}{19} \doteqdot 56.84$ ✓ — Mark

Correct value of v for stone

$\underset{\sim}{v}(t) = V\underset{\sim}{i} + (-10t)\underset{\sim}{j}$.

$\underset{\sim}{v}(t-8) = V\underset{\sim}{i} + (-10(t-8))\underset{\sim}{j}$.

$$\text{Speed} = \sqrt{v^2 + (-10(27-8))^2}$$

$$= \sqrt{\left(\frac{40 \times 27}{19}\right)^2 + (-10 \times 19)^2}$$

$= 198.32$

$\doteqdot 198$ m/s ✓ — Correct answer.